

# Private Information and Intertemporal Job Assignments

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## Abstract

This paper studies the assignment of people to projects over time in a model with private information. The combination of risk neutrality with incomplete contracts that restrict the ability of an agent to report on interim states is a force for long-term assignments. More generally, however, rotating agents can be valuable because it conceals information from agents, which mitigates incentive constraints. With complete contracts that communicate interim states, rotation allows for even more concealment possibilities and better targeted incentives. Furthermore, it allows for the reporting of interim shocks at no cost to the principal. Properties of the production technology are also shown to matter. Substitutability of intertemporal effort is a force for long-term assignments while coordination with Nash equilibrium strategies is a force for job rotation.

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# 1 Introduction

This paper studies the problem of a firm that must assign people to operate projects over time. Each project operates over more than one stage, but in any single stage the principal may assign only one person to it. The principal's information about each project is limited; he observes a project's output, but not its interim state nor its labor inputs. Only the agent assigned to a project at a particular stage observes that stage's relevant variable, be it the interim state or his own labor input. In addition to setting standard contractual terms such as output-dependent consumption, the principal has the ability to rotate agents among the projects. Providing conditions under which these reassignments occur is the goal of this paper. The conditions we examine include communication possibilities, conditions on preferences, and technological coordination.

Organizations regularly face assignment problems. Conglomerates must decide how to allocate executives across divisions. Firms must decide how to allocate managers across departments. Managers must decide how to allocate employees across jobs. Frequently, these decisions have time and contingent components. How long should a manager be assigned to a project? Under what conditions should he be rotated? Regular periodic job rotation is one strategy undertaken by many organizations. Executives are rotated across divisions, and managers are rotated across functional areas. Even within a function employees may be rotated. For example, many large banks rotate their loan officers among lending offices.<sup>1</sup> This solution to the assignment problem is costly. Job-specific knowledge is lost and time is spent learning details specific to the new assignment. Yet, despite these costs organizations still regularly rotate people.<sup>2</sup>

There are several theories of intertemporal job assignment. In Meyer (1994), varying the assignment over time of workers to teams helps an organization learn about the ability of workers. New assignments can also provide training for managers who are later promoted. In Ickes and Samuelson (1987), rotation can solve "a ratchet effect." For incentive reasons a long-term contract is beneficial, but rotation is the only way the organization can commit

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<sup>1</sup>Banks also often require certain employees to take vacations over an extended continuous period each year. The purpose of this temporary rotation is to make it harder for the employee to perpetuate a fraud.

<sup>2</sup>Osterman (2001) documents that in 1997 56% of US establishments with more than 50 employees used job rotation. Lindback and Snower (2001) list other studies that document the use of job rotation (and other work practices).

to it. In Hirao (1993) and Arya and Mittendorf (2004), rotation allows a firm to obtain information at no cost.

Our goal in this paper is to identify additional forces – complementary to those identified by the literature – that lead to rotation. The new forces we identify include information scrambling and properties of production technologies. Furthermore, we describe how information revelation can be used to better target incentives for agents working a project in later stages. Unlike Ickes and Samuelson (1987), we do not rely on limited commitment by the principal. We also explore the role that communication or, equivalently, a menu of contracts, plays.

Our models are also relevant for two other literatures. The first one is on second-sourcing in procurement problems, that is, when a procurer can switch suppliers at the later stage of the procurement process. Papers in this literature include Anton and Yao (1987), Demski, Sappington, and Spiller (1987), Riordan and Sappington (1987, 1989), and Lewis and Sappington (1997). Unlike this literature, we consider risk aversion. Furthermore, we emphasize the role of information scrambling and the coordination properties of the production function.

The second relevant literature concerns the value of information and communication in the design of accounting systems for managerial incentive purposes. Among other questions, this literature asks whether it is valuable to allow the agent to observe production information that the principal does not observe. Papers addressing this question include Christensen (1981), Penno (1984), and Baiman and Sivaramakrishnan (1991).<sup>3</sup> While related, our work also studies the value of only allowing the *principal* to know information. Furthermore, we study more general production structures.

Section 2 lays out the general environment and Sections 3 and 4 each analyze a prototype. Section 3 studies the first prototype, a two-stage model with an interim state in the first stage followed by a labor input in the second. The interim state is observed only by the agent initially assigned to the project, while the labor effort is observed only by the agent assigned to the project in the second stage. Between the two stages the agents can

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<sup>3</sup>Lewis and Sappington (1994) study a monopolist problem with varying demand where the question is similar. “Is it valuable for the monopolist to provide a signal to potential buyers of how much they will value the product?” Unlike the above literature, they do not have a second-stage moral hazard problem.

be rotated, or switched, to different projects. As a benchmark, we study the incomplete contract case where agents are not allowed to choose from menu of contracts. Switching hides information from agents so an agent cannot tailor his effort to the interim state of the new project. Still, hiding information can be beneficial because it mitigates second-stage moral-hazard constraints. Under risk neutrality, the former effect dominates so agents are not switched. An example with risk aversion is provided in which the latter effect dominates so agents are switched.

With complete contracts switching is always optimal and often strictly dominates. First, switching allows the principal to learn interim states at no cost; an agent sends a truthful report as long as his compensation does not depend on his report. Second, knowledge of the interim state allows the principal to scramble information as in the incomplete contract benchmark and target incentives to an agent's new assignment.

Section 4 studies the second prototype, a model in which the interim state stage of the first prototype is replaced by one in which the agent takes a hidden effort. We find that the optimality of switching depends on the substitution and coordination properties of production over the two project stages. Substitution is a force for long-term assignment, as the agent takes full responsibility for all stages of production effort. Coordination is a force for switching because the resulting Nash equilibrium in efforts alleviates incentive constraints.

Section 5 returns to the generalized model and discusses it. Section 6 incorporates some concluding comments. The Appendix contains a proof.

## 2 The Environment

There is a continuum of agents and a continuum of projects, both of measure one. The continuum assumption should be viewed as an approximation to the large number of people and projects that make up a firm. This abstraction avoids the need to worry about aggregate uncertainty that may arise when there is a finite number of agents and shocks are identically and independently distributed.

Production on a project takes multiple stages. In the first stage there is an action  $a$  on each project. This action determines the probability distribution of an interim state  $\theta$ .

The conditional probability distribution is  $h(\theta|a)$ . The state  $\theta$  can only take on a finite number of realizations. Shocks are independent across projects, but with the continuum assumption  $h(\theta|a)$  can also be viewed as the fraction of projects experiencing state  $\theta$  given  $a$ . In the second stage of production, each project requires a labor input  $b \in B$ . On each project, the state and the labor input determine the conditional probability distribution of the project's output  $q \in Q$ ; the set  $Q$  is finite. We write the conditional distribution as  $p(q|a, b, \theta)$ . The state  $\theta$  of a project and the labor input  $b$  applied to it do not affect production on any other project.

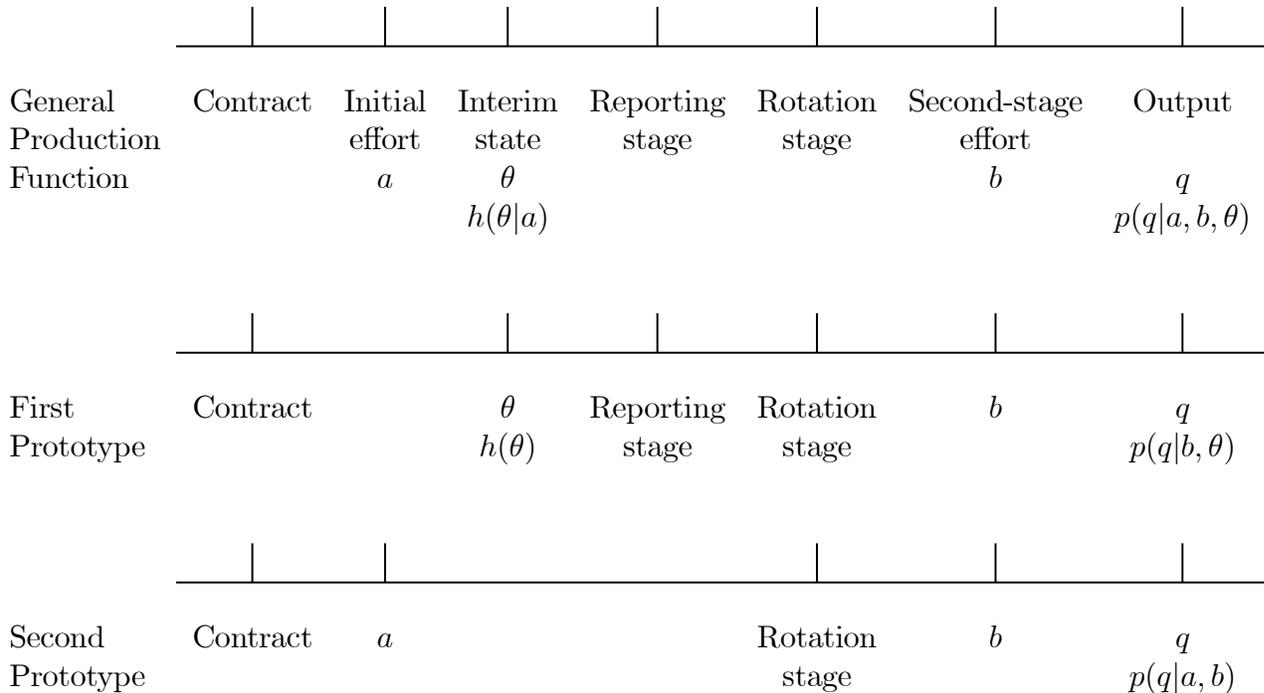


Figure 1: Time lines for three multi-stage production functions. The top time line is for the most general case. The first prototype drops the initial effort. The second prototype drops the interim state. Agents can only be switched between projects at the rotation stage. Switching is a choice variable of the principal. The reporting stage is when an agent can communicate his shock to the principal, effectively choosing from a menu of contracts.

In each stage only one agent may be assigned to a particular project. Each agent starts out assigned to an initial project. This agent takes the initial action  $a$  on his initial project. This action is private information. After the initial action, the interim state  $\theta$  of a project is realized. This state is also private information. It is *only* observed by the agent assigned

to that project at the time of realization. In the most general version of the model, the agent may report the interim state to the principal, that is, choose from a menu of contract. After this reporting, if any, the principal may assign agents to new projects. In the final stage of production, the agent assigned to a project supplies the labor input  $b$ , which is also private information. If assigned to a new project for this last stage, he will neither know the interim state  $\theta$  nor the initial recommended labor input  $a$  unless the principal tells him. Finally, output produced on each project is public information. The first time line in Figure 1 illustrates the different stages of production for a project. It also illustrates when switching, if any, occurs.

An agent's preferences are

$$U(c) - V(a) - V(b),$$

where  $c$  is his consumption, with  $c \in \mathfrak{R}_+$ . We assume that the functions  $U$  and  $V$  are increasing, that  $U$  is weakly concave, and that  $V$  is weakly convex. At the time of contracting, each agent has an outside option that gives him  $\bar{U}$  utils. Once he agrees to a contract, he cannot leave it. The principal, or firm, is risk neutral.

Because all projects are ex ante identical, the initial assignment of agents does not matter. Any agent can be assigned to any project. The interesting assignment problem occurs later, during the stages of production. To simplify the analysis we will consider two simplifications of the general production function in Figure 1. In the first prototype, there is no initial action. Only the interim state  $\theta$  and the second stage effort  $b$  affect production. In this prototype, we also study the effect of allowing the agent to report on the interim state to the principal and focus on whether or not to switch the agents at this point. The different stages are illustrated by the second time line in Figure 1.

In the second prototype, we shut down the interim state so production is only determined by the effort inputs  $a$  and  $b$ . Here, we analyze whether it is desirable to switch the agents between the two stage of production. The timing for this prototype is illustrated by the third time line in Figure 1.

### 3 First Prototype

In this section, we consider the first prototype, the second time line in Figure 1. In this model an agent observes the interim state of his project and then may be switched to a new project. One important feature of these contracts is whether or not an agent communicates the interim state to the principal or, equivalently, chooses from a menu of contracts. In the following subsection, we follow the tradition of the incomplete contracts literature and assume that contracts cannot be made contingent on the interim state. This case provides a useful benchmark. Afterwards, we study the complete and information-constrained contract that allows unrestricted communication possibilities and compare and contrast the results of the two approaches. For convenience, we assume in this section that  $B$  is finite.

#### 3.1 Incomplete contract benchmark

In this section, we assume that states  $\theta$  in the first stage are private to the agent initially assigned to the project, and that effort  $b$  is private to the agent assigned to the project in the second stage. Furthermore, we assume that an agent *can neither* report the state of his initial project *nor* choose from a menu of contracts. Despite this limitation in the contract, the principal may still switch agents across projects after the first stage. If he does this, the reassignment must, by necessity, be random across project states  $\theta$ .

We analyze this problem by separately considering two regimes. One in which agents stay on their initial project and another in which they are rotated and randomly assigned to a new project. We then compare the two programs to determine which regime is optimal.

We start with the no-switching regime. In this regime, each agent stays on his initial project so he knows the interim state  $\theta$ . A *no-switching incomplete contract* is a state-contingent effort recommendation  $b(\theta)$  and a consumption sharing rule  $c(q)$ . The effort recommendation is of the form, “If you receive state  $\theta$  then take effort level  $b(\theta)$ .” The sharing rule only depends on  $q$  because the principal neither observes nor receives a report on the interim state. There will be a set of incentive constraints guaranteeing that  $b(\theta)$  is incentive compatible for each  $\theta$ .

The programming problem is

*Program 1:*

$$\begin{aligned}
& \max_{c(q), b(\theta)} \sum_{\theta} h(\theta) \sum_q p(q|b(\theta), \theta)(q - c(q)) \\
\text{s.t. } & \sum_{\theta} h(\theta) \sum_q p(q|b(\theta), \theta)(U(c(q)) - V(b(\theta))) \geq \bar{U}, \tag{1}
\end{aligned}$$

$$\sum_q p(q|b(\theta), \theta)(U(c(q)) - V(b(\theta))) \geq \sum_q p(q|\hat{b}, \theta)(U(c(q)) - V(\hat{b})), \forall \theta, \forall \hat{b} \neq b(\theta). \tag{2}$$

The objective function is the expected utility of the principal. From the principal's perspective  $h(\theta)$  is the fraction of type- $\theta$  projects he has. Equation (1) is the ex ante participation constraint for each agent. From an agent's perspective,  $h(\theta)$  is the probability he works a type- $\theta$  project. Equation (2) represents the incentive constraints referred to earlier.

If agents are switched, neither the principal *nor the agent newly assigned to a project* know its interim state. Consequently, all agents must be assigned the same effort level  $b$ . A *switching incomplete contract* is an effort level  $b$  and a consumption schedule  $c(q)$ .

The problem if agents are switched is

*Program 2:*

$$\begin{aligned}
& \max_{c(q), b} \sum_{\theta} h(\theta) \sum_q p(q|b, \theta)(q - c(q)) \\
\text{s.t. } & \sum_{\theta} h(\theta) \sum_q p(q|b, \theta)(U(c(q)) - V(b)) \geq \bar{U}, \tag{3}
\end{aligned}$$

$$\sum_{\theta} h(\theta) \sum_q p(q|b, \theta)(U(c(q)) - V(b)) \geq \sum_{\theta} h(\theta) \sum_q p(q|\hat{b}, \theta)(U(c(q)) - V(\hat{b})), \forall \hat{b} \neq b. \tag{4}$$

Again, the objective function is the principal's utility function. Equation (3) is the ex ante participation constraint for each agent. Equation (4) is the incentive constraint. Notice that unlike in the no-switching program, there is not one set of incentive constraints for each  $\theta$ . Instead, because each agent does not know the  $\theta$  of his newly assigned project, he must form the expectation using as a prior the distribution  $h(\theta)$  in the population.

Each regime has an advantage and a disadvantage relative to the other. If agents are not switched, their efforts can be tailored to the relative productivities of each project, but there

are more incentive constraints. If agents are switched there are less incentive constraints but the same effort is applied to all projects, regardless of relative productivities. As the next proposition demonstrates, risk neutrality is an important factor in determining which regime is better.

**Proposition 1** *If agents are risk neutral and consumption can be negative, then no-switching weakly dominates switching.*

**Proof:** Make each agent the residual claimant and make him pay a constant amount such that his participation constraint holds. This aligns each agent’s incentives with that of the principal. **Q.E.D.**

If consumption is restricted to be non-negative then this result need not hold. The lower bound on consumption can sometimes interfere with perfectly aligning the incentives. Still, in general, risk neutrality is a force for no switching. Without the incentive distortion, the ability to tailor effort levels to marginal productivities is unambiguously good.

If there is an incentive distortion, either because of these lower bound issues in the risk neutrality case or for other reasons, switching may dominate. In particular, the agent’s resulting ignorance of the interim state can relax incentive constraints. The following example shuts down the value of varying effort  $b$  with  $\theta$  to illustrate the value of relaxing the incentive constraints.

### 3.1.1 Example 1

Agents may choose from only three possible efforts,  $b_1, b_2,$  or  $b_3$ . The effort portion of the utility function is described by  $V(b_1) = V(b_2) < V(b_3)$ . There are two different types of projects, indexed by  $\theta_1,$  and  $\theta_2$ . Project types are random and drawn from the distribution  $h(\theta_1) = h(\theta_2) = 0.5$ . Each type of project may produce either a low output,  $q_l$  or a high output  $q_h$ . Output on each project is independent of other projects. Table 1 describes the  $p(q|b, \theta)$  production function used in the example.

The two types of projects are identical except that  $b_i, i = 1, 2$  has a different effect on each project. If  $b_i$  is worked on a  $\theta_i, i = 1, 2,$  project then the project is extremely unproductive.

		$\theta_1$		$\theta_2$	
		$q_l$	$q_h$	$q_l$	$q_h$
$b_1$		1.00	0.00	$b_1$	0.80 0.20
$b_2$		0.80	0.20	$b_2$	0.00 1.00
$b_3$		0.40	0.60	$b_3$	0.40 0.60

Table 1: A production technology,  $p(q|b, \theta)$ , that generates switching.

First, we consider an arbitrary no-switching contract in which the planner wants to implement the high disutility of effort  $b_3$  on both projects so  $c(q_l) < c(q_h)$ . For the agent assigned to the  $\theta_1$  project, there are two incentive constraints. The first one prevents deviating to  $b_1$  and the second one prevents deviating to  $b_2$ . They are

$$\begin{aligned}
0.4U(c(q_l)) + 0.6U(c(q_h)) - V(b_3) &\geq U(c(q_l)) + 0U(c(q_h)) - V(b_1) \\
0.4U(c(q_l)) + 0.6U(c(q_h)) - V(b_3) &\geq 0.8U(c(q_l)) + 0.2U(c(q_h)) - V(b_2). \quad (5)
\end{aligned}$$

The incentive constraints for the agent assigned to the  $\theta_2$  project are nearly identical. They are

$$\begin{aligned}
0.4U(c(q_l)) + 0.6U(c(q_h)) - V(b_3) &\geq 0.8U(c(q_l)) + 0.2U(c(q_h)) - V(b_1) \quad (6) \\
0.4U(c(q_l)) + 0.6U(c(q_h)) - V(b_3) &\geq U(c(q_l)) + 0U(c(q_h)) - V(b_2),
\end{aligned}$$

for  $b_1$  and  $b_2$ , respectively. For both pairs, the binding incentive constraint is the one with the 80% chance of the low output and the 20% chance of the high output. This strategy always dominates the strategy of producing the low output with certainty.

The principal can do better with a switching contract. If the agents are switched, they do not know the interim state of their new project. All they know is that there is a 50% chance they were assigned to each type of project. Now, if the principal wants to implement the  $b_3$  action, the incentive constraints preventing  $b_1$  and  $b_2$  are

$$0.4U(c(q_l)) + 0.6U(c(q_h)) - V(b_3) \geq 0.9U(c(q_l)) + 0.1U(c(q_h)) - V(b_1) \quad (7)$$

$$0.4U(c(q_l)) + 0.6U(c(q_h)) - V(b_3) \geq 0.9U(c(q_l)) + 0.1U(c(q_h)) - V(b_2), \quad (8)$$

respectively.

The value of deviating is the convex combination of the effect of working on both projects. The Bayesian updating from being assigned to each type of project with a 50% chance does not change the utility from taking the recommended action (the left-hand side of the incentive constraints), but it does change the utility from deviating. In particular, by lowering the utility relative to the best alternative available to the agent if he knew the state, the principal has lowered the value of deviating. This can be seen formally by comparing the binding no-switching incentive constraints, (5) and (6), with the switching incentive constraints, (7) and (8). Each of the two switching incentive constraints is a convex combination of two of the four no-switching incentive constraints. Therefore, allocations that satisfy the no-switching constraints always satisfy the switching constraints but not vice versa.

### 3.2 Information-constrained complete contracts

In this section, we place no restrictions on the use of reports of  $\theta$  in the contractual terms. Formally, this allows the consumption schedules to be indexed by  $\theta$ , that is, we allow the agents to report, or equivalently to choose from a menu of contracts, after observing  $\theta$ . Papers by Demougin (1989), Melamud and Reichelstein (1989), and Penno (1984) have demonstrated that incorporating  $\theta$  into the no-switching model can be valuable. In this section, we will see that the combination of reporting on  $\theta$  and switching the agents is powerful: it allows for the costless revelation of information to the principal. Furthermore, the principal can use the information to make scrambling of information more effective than in the incomplete contracts case.

### 3.3 Information revelation

For reasons illustrated shortly, we allow for some randomization in contractual terms. As in the no-switching model, the contract contains a recommended effort level that depends on the interim state of the project. Now, however, this recommendation may be random. It is described by the conditional probability distribution  $\pi(b|\theta)$ . Because of the randomized effort, consumption needs to be a function not only of the interim state  $\theta$  and the output realization  $q$ , but also the realized recommended effort  $b$ . We write the compensation

schedule as  $c(q, b, \theta)$ . A *no-switching contract with communication*, that is, with a menu of contracts, is a possibly random recommended effort level  $\pi(b|\theta)$  and a consumption sharing rule  $c(q, b, \theta)$ .

We first consider no-switching contracts. When an agent stays on his project with probability one, he knows the state  $\theta$ . By the Revelation Principle the contract needs to satisfy incentive constraints that induce truthful reporting of  $\theta$  and then, given a truthful report, other constraints that ensure that the agent takes the recommended effort. The truth-telling constraints are

$$\begin{aligned} & \forall \theta, \sum_{q,b} p(q|b, \theta) \pi(b|\theta) [U(c(q, b, \theta)) - V(b)] \\ & \geq \sum_{q,b} p(q|\phi(b), \theta) \pi(b|\theta') [U(c(q, b, \theta')) - V(\phi(b))], \quad \forall \theta' \neq \theta, \forall \phi : B \rightarrow B. \end{aligned} \tag{9}$$

Constraints (9) ensure that telling the truth,  $\theta$ , and then taking the resulting recommended effort  $b$ , is preferable to lying, *i.e.*, sending a report  $\theta' \neq \theta$ , and then taking any deviation strategy,  $\phi(b)$ , which maps recommended effort  $b$  to alternative effort  $b'$ . For more details on these constraints, see Myerson (1982) for the original treatment or Prescott (2003) for an exposition in a similar model.

In addition to constraints (9), the Revelation Principle requires constraints that ensure that an agent who truthfully reports  $\theta$  takes recommended effort  $b$ .<sup>4</sup> These are

$$\forall \theta, b \ni \pi(b|\theta) > 0, \sum_q p(q|b, \theta) [U(c(q, b, \theta)) - V(b)] \geq \sum_q p(q|\hat{b}, \theta) [U(c(q, b, \theta)) - V(\hat{b})], \quad \forall \hat{b}. \tag{10}$$

With complete contracts a strikingly simple mechanism improves upon no-switching contracts.

**Proposition 2** *Switching and telling the agent the state of his newly assigned project  $\theta$  weakly dominates not switching him. Dominance is strict if incentive constraints (9) bind.*

**Proof:** Consider the following contract: After agents report on their interim states, the principal switches them and makes their new assignment and compensation independent of

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<sup>4</sup>With a minor change in notation, we could have incorporated these incentive constraints with the truth-telling incentive constraints (9). We did not do this because in the following analysis it is useful to separate them.

the report they sent. Under this contract, an agent's utility does not depend on his report so he reports the true state. Next, assume that the quality of each agent's assigned project is randomly drawn from the distribution  $h(\theta)$  and the principal tells each agent the quality of his new project  $\theta$ . The compensation schedule is still described by  $c(q, b, \theta)$ , but now  $\theta$  is the actual quality of an agent's *newly* assigned project.

Because the principal and the agent know the state of the project, there are no truth-telling constraints as in equation (9). The only incentive constraints left are those on the agent's effort, which are identical to constraints (10) in the no-switching scheme. Thus, the set of no-switching contracts is a subset of the switching contracts. Consequently, switching weakly dominates no-switching. The dominance is strict if truth-telling constraints (9) bind in the no-switching regime, as these are eliminated in the switching regime. **Q.E.D.**

In the first-stage of the switching scheme, agents are simply information monitors. They report the true state because they are indifferent to what they observe and what they report.<sup>5</sup> The arrangement is essentially a moral-hazard economy, with the added feature that there is a random, publicly observed shock to the production technology.

### 3.4 Information scrambling and knowledge of $\theta$

In the contract described above the principal tells the agent the interim state of his newly assigned project. That property of the contract was imposed by fiat. While sufficient to illustrate the information revelation role of switching, it need not be optimal. Indeed, sometimes the principal would choose *not* to tell the agent the state of his newly assigned project. In this case, not only does switching remove truth-telling constraints but it also scrambles information. The agent now has to infer the quality of his newly assigned project. As we saw earlier, scrambling can weaken second-stage incentive constraints. As we will see below, letting the principal know  $\theta$  expands the opportunities for the principal to use scrambling.

When agents are switched, the new assignments must respect the supply of each type of projects. The supply of each type- $\theta$  project is  $h(\theta)$ . Because all agents are ex ante identical, the best such assignment is the random one  $h(\theta)$ . As before, the principal knows  $\theta$  and

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<sup>5</sup>A similar idea is used in Hirao (1993) and Arya and Mittendorf (2004).

recommends an effort level  $b$  according to the possibly stochastic rule  $\pi(b|\theta)$ . Therefore, we can define a *switching contract with communication* as a possibly random recommended effort level  $\pi(b|\theta)$  and a consumption sharing rule  $c(q, b, \theta)$ .

The only difference from the no-switching contract is that each agent no longer knows the  $\theta$  of his project. Since the principal does not directly tell the agent the quality of his new project, the agent has to infer it. He has two pieces of information from which to form his inference: the assignment rule  $h(\theta)$ , and the recommended effort rule  $\pi(b|\theta)$ . An agent who is recommended effort  $b$  forms a posterior over project quality of  $pr(\theta|b)$ . The posterior is related to the other objects by the relationship

$$pr(\theta|b) = h(\theta)\pi(b|\theta)/\pi(b), \tag{11}$$

where  $\pi(b)$  is the unconditional probability that an agent is recommended effort  $b$ .

The incentive constraint can be written directly in terms of the posterior probabilities,  $pr(\theta|b)$ , but it is more convenient to substitute out for these terms. Again, there are no truth-telling constraints, only moral hazard constraints. These constraints are: for all  $b$  such that  $\pi(b) > 0$ ,

$$\sum_{q,\theta} p(q|b, \theta)\pi(b|\theta)h(\theta)[U(c(q, b, \theta)) - V(b)] \geq \sum_{q,\theta} p(q|\hat{b}, \theta)\pi(b|\theta)h(\theta)[U(c(q, b, \theta)) - V(\hat{b})], \quad \forall \hat{b} \neq b, \tag{12}$$

where the  $\pi(b)$  in equation (12) cancels out of both sides.

Compare these moral hazard constraints, (12), with the moral hazard constraints, (10), used by the other two schemes. For a given  $b$ , (12) is a convex combination of all the incentive constraints (10) corresponding to  $\theta$  for which  $b$  was recommended. We can now prove the following theorem.

**Proposition 3** *A switching contract where the principal does not tell the agent the state  $\theta$  of his newly assigned project weakly dominates a switching contract where the principal tells the agent the value of  $\theta$  of his newly assigned project.*

**Proof:** Any contract satisfying (10) for each  $\theta$  will satisfy (12), but not necessarily vice versa. **Q.E.D.**

For Proposition 3 to hold strictly it is necessarily to have a problem in which agents assigned to different projects are still recommended the same effort a positive fraction of the time. Otherwise, if each agent assigned to a different quality project was recommended a different effort level then each agent would perfectly infer the quality of his new project from the effort recommendation. There would not be any scrambling and no reason for the principal to hide information from the agent.

The logic is exactly the same as that used in Example 1 in the incomplete contracts case. The difference is that now consumption and effort levels can directly depend on  $\theta$ . Consequently, efforts can still be tailored to project productivities, and compensation sharing rules can be indexed by the assignment. The numerical example below demonstrates the value of the latter feature.

### 3.4.1 Example 2

In Example 1, the principal did not need to know the state of a project in order to generate the desired amount of scrambling. In the following example, the principal wants to know the state because it will help him make an inference about effort from the observed output.<sup>6</sup> In terms of the notation, there is value to indexing the compensation schedule by the  $\theta$  the agent is assigned to.

We will use the same technology as above, but now effort and the state of the project also produce one of two signals,  $s_1$  and  $s_2$ . We follow Holmstrom (1979) in that these signals do not have any direct effect on output. Furthermore, to simplify the analysis, we make the signals independent of output and thus write their conditional probability distribution as  $f(s|b, \theta)$ . The probability distribution of output,  $p(q|b, \theta)$ , is the same as that of Table 1 in Example 1. Table 2 illustrates a possible distribution of signals.

If the principal does know  $\theta$ , as he would under the revelation mechanism developed earlier, then the signal conveys information about whether an agent took the recommended action  $b_3$ . For example, an agent assigned to a  $\theta_1$  project and who takes  $b_1$  or  $b_2$ , is less likely to produce signal  $s_1$  and more likely to produce  $s_2$  than an agent who takes  $b_3$ . Consequently, the principal will want to index consumption by the signal. This same agent

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<sup>6</sup>Knowledge of  $\theta$  would also allow the principal to choose the amount of scrambling, as might be desirable in a model with heterogeneous agents. See Prescott and Townsend (1993) for such an example.

		$\theta_1$		$\theta_2$	
		$s_1$	$s_2$	$s_1$	$s_2$
$b_1$		0.50	0.50	$b_1$	0.50 0.50
$b_2$		0.50	0.50	$b_2$	0.50 0.50
$b_3$		0.90	0.10	$b_3$	0.10 0.90

Table 2: A production technology for signals,  $f(s|b, \theta)$ , for which it is valuable for the principal to know  $\theta$ .

will be rewarded more for producing the high output and signal  $s_1$  than if he produces the high output and signal  $s_2$ . The scenario is reversed if an agent is assigned to a  $\theta_2$  project.

## 4 Second Prototype

In the previous sections, the quality of a project was determined by a random shock. With communication the resulting interim state could be elicited at no cost by the principal. An agent’s role in the first stage of a production process was simply to gather information. There are many situations, however, where the quality of a project would be determined by the efforts taken by an agent. In this section, we study this question by replacing the interim state  $\theta$  in the first stage with an initial effort level  $a$ , as in the third time line in Figure 1. This effort level is taken by the agent initially assigned to a project and is private information to him. As with the interim state, an agent assigned to a *new* project in the second stage does not observe the effort  $a$  taken on it by the initial agent and is induced the recommended second stage effort  $b$ , regardless of effort  $a$ . We focus our analysis on coordination in production between the two efforts.

To keep this problem tractable, we restrict our analysis to symmetric contracts. We also assume that the sets  $A = B \subset \mathfrak{R}_+$ . Output on a particular project is a function solely of efforts  $a$  and  $b$  taken on that project and a project-specific random shock. These latter shocks across projects are uncorrelated. Specifically, if agents are *not* switched, the production function on an agent’s project is written simply as  $p(q|a, b)$ , where  $p$  denotes the probability of output  $q$  given efforts  $a$  and  $b$  in the two stages. If agents are switched, then it is necessary to keep track of output on both projects to which an agent was assigned. The production function from his perspective is  $p(q_1|a, b^*)p(q_2|a^*, b)$ , where  $a^*$  and  $b^*$  are

the first and second stage efforts recommended to others, either before he arrives or after he leaves, respectively. We adopt the convention that from the agent's perspective,  $q_1$  refers to his initial project whether or not he is switched and  $q_2$  refers to his second-stage project if he is switched from his original project. Similarly, the consumption of an agent who is not switched is  $c(q_1)$ , while the consumption of a switched agent is  $c(q_1, q_2)$ .

Agents receive utility from consumption and disutility from efforts. Utility is separable and written  $U(c) - V(a) - V(b)$ , where  $U$  is strictly concave and  $V$  is strictly convex.

For analytical reasons we make several simplifications. First, we model switching as a discrete decision made by the principal at the time of contracting. Thus, all participants in the economy know beforehand whether or not they will be switched. Our second simplification is to allow the principal to only send messages (recommending efforts) immediately after contracting and not at an interim stage. This assumption precludes the principal from recommending an interim-stage effort  $a$  and then, after the interim stage effort is taken, sending a random message which recommends a final-stage effort  $b$ .<sup>7</sup> Finally, as noted, we also restrict our focus to symmetric contracts. Symmetry here means that agents face the same contract *and* are recommended the same sequence of efforts. Essentially, our framework reduces the incentive constraints to a symmetric, pure strategy, Nash equilibrium in the game played between agents. However, realizations of output may still differ across projects.

We start by considering the no-switching contract. The optimal no-switching contract solves

*Program 3:*

$$\max_{c(q_1), a, b} \sum_{q_1} p(q_1|a, b)(q_1 - c(q_1))$$

$$\text{s.t. } \sum_{q_1} p(q_1|a, b)U(c(q_1)) - V(a) - V(b) \geq \bar{U}, \quad (13)$$

$$\sum_{q_1} p(q_1|a, b)U(c(q_1)) - V(a, b) \geq \sum_{q_1} p(q_1|\hat{a}, \hat{b})U(c(q_1)) - V(\hat{a}) - V(\hat{b}), \forall \hat{a} \neq a, \hat{b} \neq b. \quad (14)$$

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<sup>7</sup>Examples can be generated where such a strategy is beneficial. Unfortunately, it greatly complicates the analysis of the switching problem.

Equation (13) is the participation constraint and equation (14) represents the incentive constraints.

If agents are switched the problem changes. Now consumption can depend on output of the two projects where an agent has worked. Since the treatment of the agents is symmetric, we can keep the problem relatively simple by representing it as a single-agent problem. The optimal switching contract solves

*Program 4:*

$$\max_{c(q_1, q_2), a, b} \sum_{q_1, q_2} p(q_1|a, b)p(q_2|a, b)(q_1 - c(q_1, q_2)) \quad (15)$$

$$\text{s.t. } \sum_{q_1, q_2} p(q_1|a, b)p(q_2|a, b)U(c(q_1, q_2)) - V(a) - V(b) \geq \bar{U}, \quad (16)$$

$$\begin{aligned} & \sum_{q_1, q_2} p(q_1|a, b)p(q_2|a, b)U(c(q_1, q_2)) - V(a) - V(b) \quad (17) \\ & \geq \sum_{q_1, q_2} p(q_1|\hat{a}, b)p(q_2|a, \hat{b})U(c_1(q_1, q_2)) - V(\hat{a}) - V(\hat{b}), \quad \forall \hat{a} \neq a, \hat{b} \neq b. \end{aligned}$$

Equation (16) is the participation constraint. The incentive constraints (17) reflect the ability of the agent to affect output on his initial project through effort  $a$  and output on his second project through effort  $b$ . If the agent deviates on his initial project he takes effort  $\hat{a}$ , which affects output on project one. Furthermore, agent one takes the subsequent effort,  $b$ , of the other agent assigned to his initial project as given so the probability distribution of output on project one is described by  $p(q_1|\hat{a}, b)$ . Similarly, when the agent contemplates deviating on his second project to  $\hat{b}$ , he takes the equilibrium initial effort of the other agent assigned to his second project as given so the probability distribution of output on project two is  $p(q_2|a, \hat{b})$ .

One unusual feature of this program is that the only output entering the objective function (15) is  $q_1$  even though consumption depends on  $q_1$  and  $q_2$ . Intuitively, some kind of formulation like this is needed because under switching, two different agents work on a given project but there is really only one project per agent and we need to avoid double counting of output. Since the projects are identical and all agents are assigned the same effort levels, it is sufficient to just use  $q_1$  rather than an average of  $q_1$  and  $q_2$ .

## 4.1 Substitutes

We now consider an environment in which the efforts  $a$  and  $b$  are perfect substitutes in the production function, that is, on each project the probability distribution of output is described by  $p(q|a+b)$ . It is sometimes useful to write total effort as  $e \equiv a+b$ . Let  $p_e(q|e)$  be the derivative of the probability of  $q$  with respect to  $e$ . A probability function  $p(\cdot)$  satisfies the *monotone likelihood ratio condition* (MLRC) if  $p_e(q|e)/p(q|e)$  is nondecreasing in  $q$ . Let  $P(q|e)$  be the corresponding cumulative distribution function and  $P_{ee}(q|e)$  the second derivative with respect to  $e$ . Then,  $P(\cdot)$  satisfies the *convexity of the distribution function condition* (CDFC) if  $P_{ee}(q|e) \geq 0$  for all  $q$  and  $e$ . In single-agent moral-hazard problems, these assumptions are sufficient for the use of the first-order approach to incentive constraints. That approach will be used in the proof.

**Proposition 4** *If efforts are perfect substitutes in production and the production function satisfies MLRC and CDFC then the optimal no-switching symmetric contract strictly dominates all switching symmetric contracts.*

**Proof:** If an agent is switched then he faces the option of deviating on project one, project two, or both. Consider any contract  $(c(q_1, q_2), a, b)$  that satisfies the first-order conditions to the agent's problem in the switching regime. These conditions are:

$$\sum_{q_1, q_2} p_e(q_1|a+b)p(q_2|a+b)U(c(q_1, q_2)) - V'(a) = 0, \quad (18)$$

$$\sum_{q_1, q_2} p(q_1|a+b)p_e(q_2|a+b)U(c(q_1, q_2)) - V'(b) = 0. \quad (19)$$

The first-order approach to incentive problems is not necessarily sufficient in the switching case. Nevertheless, these conditions are still necessary for a solution and that is all we need for our proof. Our strategy is to show that for any contract satisfying a relaxed switching program, that is, (18) and (19), we can construct a better, incentive compatible, no-switching contract.

If the switching contract is characterized by  $a = b$  then the proof can skip to (24) to construct a better no-switching contract. If, instead, it is characterized by  $a \neq b$  then the following steps need to be taken first. Consider a second contract,  $(\bar{c}(q_1, q_2), \bar{a}, \bar{b})$ ,

that satisfies (18) and (19) and is the mirror image of  $(c(q_1, q_2), a, b)$ . Specifically, let  $\bar{c}(q_1, q_2) = c(q_2, q_1)$ ,  $\bar{a} = b$ , and  $\bar{b} = a$ . Notice that since  $a \neq b$  then  $c(q_1, q_2) \neq \bar{c}(q_1, q_2)$  for some pairs of outputs. Because the two contracts are mirror images, both give the principal and the agent the same utility. The first-order condition on initial effort is

$$\sum_{q_1, q_2} p_e(q_1|\bar{a} + \bar{b})p(q_2|\bar{a} + \bar{b})U(\bar{c}(q_1, q_2)) - V'(\bar{a}) = 0.$$

Substituting in for the effort levels, this condition implies that

$$\sum_{q_1, q_2} p_e(q_1|a + b)p(q_2|a + b)U(\bar{c}(q_1, q_2)) - V'(b) = 0. \quad (20)$$

The average effort level for both contracts is  $(a + b)/2$ . Because  $V'((a + b)/2)$  is between  $V'(a)$  and  $V'(b)$ , equations (18), (20), and continuity imply that there exists  $\alpha \in (0, 1)$  such that

$$\sum_{q_1, q_2} p_e(q_1|a + b)p(q_2|a + b)[\alpha U(c(q_1, q_2)) + (1 - \alpha)U(\bar{c}(q_1, q_2))] - V'((a + b)/2) = 0. \quad (21)$$

Now construct another contract by leaving effort levels unchanged and setting consumption to satisfy

$$c'(q_1, q_2) = U^{-1}(\alpha U(c(q_1, q_2)) + (1 - \alpha)U(\bar{c}(q_1, q_2))), \quad (22)$$

where  $U^{-1}$  is the inverse of the utility function  $U$ . Then, substituting  $U(c'(q_1, q_2))$  into (21) gives

$$\sum_{q_1, q_2} p_e(q_1|a + b)p(q_2|a + b)U(c'(q_1, q_2)) - V'((a + b)/2) = 0. \quad (23)$$

The contract  $(c'(q_1, q_2), (a + b)/2, (a + b)/2)$  that satisfies (22) and (23) gives the agent more utility than the initial switching contract because of the convexity of  $V$ , and it gives the principal more surplus because of the concavity of  $U$ . It need not satisfy (19) so it might not even be feasible to the relaxed switching program. Still, it is better than our initial switching contract so if we can find an even better no-switching contract then that no-switching contract must be better than the initial switching contract, too.

Consider the no-switching contract  $(\tilde{c}(q_1), (a + b)/2, (a + b)/2)$  that satisfies

$$\tilde{c}(q_1) = U^{-1}\left(\sum_{q_2} p(q_2|a + b)U(c'(q_1, q_2))\right), \quad \forall q_1, \quad (24)$$

where, again,  $U^{-1}$  is the inverse of the utility function  $U$ . Substitution of  $U(\tilde{c}(q_1))$  into (23) delivers

$$\sum_{q_1} p_e(q_1|a+b)U(\tilde{c}(q_1)) - V'((a+b)/2) = 0. \quad (25)$$

Let  $\tilde{V}(e) = \min_{a,b} V(a) + V(b)$  subject to  $a+b = e$ . This object is the indirect disutility of effort received by the agent. By the Theorem of Maximum (actually minimum here)  $\tilde{V}$  is a convex function like  $V$ . Because of the symmetry, any solution to the agent's problem will be characterized by  $a = b = e/2$ , which implies that  $\tilde{V}'(e) = V'(e/2)$ . Substituting into (25) delivers

$$\sum_{q_1} p_e(q_1|e)U(\tilde{c}(q_1)) - \tilde{V}'(e) = 0. \quad (26)$$

This is the first-order condition to the agent's problem in the no-switching regime, just expressed in terms of total effort  $e$ . Furthermore, the first-order approach is sufficient for the no-switching problem because of the assumptions of MLRC and CDFC (see Rogerson (1985) or Hart and Holmstrom (1987)). Therefore, the contract  $(\tilde{c}(q_1), a, b)$  is incentive compatible in the no-switching regime. By construction, it gives the agent the same utility as the  $(c'(q_1, q_2), (a+b)/2, (a+b)/2)$  contract and it gives the principal more utility because concavity of  $U$  implies that  $\tilde{c}(q_1) < \sum_{q_2} p(q_2|a+b)c'(q_1, q_2)$ . Furthermore, since that contract is, in turn, better than the initial switching contract, no switching strictly dominates switching. **Q.E.D.**

No-switching contracts are powerful in this model because they allow incentives to be focused on one project rather than two, thereby reducing consumption variation.

## 4.2 Coordination

In contrast, switching can be valuable for production functions that require coordination in the two inputs. For this result, we consider the production function  $p(q|f(a, b))$ , where coordination is expressed through the functional form of  $f(a, b, )$ . Specifically,

**Assumption 1** *The function  $f(a, b)$  is Leontief, that is,  $f(a, b) = \min\{a, b\}$ .*

The next proposition provides conditions under which switching strictly dominates no switching.

**Proposition 5** *If  $f(a, b)$  satisfies Assumption 1, then the best switching contract strictly dominates the best no-switching contract.*

**Proof:** See Appendix.

When an agent is switched, he plays a Nash-like game in efforts with the other agents assigned to his projects. As a consequence, all of his feasible deviations push him to off-diagonal effort pairs, which are relatively unproductive. In contrast, an agent who is not switched can deviate in both stages to achieve the same level of drop in productivity while reducing the disutility of effort by twice as much. Consequently, deviations under a no switching regime are more expensive to prevent.

The coordination case is similar to the problems studied in the team literature, e.g. Holmstrom (1982) and Legros and Mathews (1993). While that literature usually has no uncertainty over output, the agents still play a Nash game in their efforts and a deviation by a single agent has a large effect on the team's output. There the problem is to get the agents to implement the optimal actions. In our model, by rotating agents the principal endogenously forms production teams. Whether they are formed depends on the relative ease of implementing actions in a team compared with implementing actions for individuals who each work alone.

## 5 Discussion of the General Model

We have isolated several forces that matter for job rotation. One force for *not* switching agents is the combination of risk neutrality with incomplete contracts that in effect limit communication from the agent to the principal, as in Section 3.1. A second force for not switching agents is substitutability in the production, as in Section 4.1. Likewise, we isolated several forces for assigning agents to new projects: incentives to truthfully report states, design of scrambling mechanisms to mitigate the moral hazard problem, and coordination in productive inputs.

The various forces for and against switching that we identified in the two prototypes may operate simultaneously. In the general model illustrated by Figure 1, assessing the relative strengths of these forces is difficult. One tractable parameterization is the special case where the second-stage production technology is  $q = b\theta$ . If agents are never switched,

there are lots of incentive constraints; some on initial effort  $a$ , some on the report of  $\theta$ , and others on second-stage effort  $b$ . But if agents are switched, then they can be played off against each other to make both the interim state  $\theta$  and the second-stage effort  $b$  public information. In equilibrium, the agent initially assigned to a project truthfully reports on  $\theta$  and the agent switched to that project takes the recommended action  $b(\theta)$  so output will be  $q = b(\theta)\theta$ . If either agent deviates, that is, the first agent misrepresents  $\theta$  or the second agent deviates from  $b(\theta)$ , then output will not equal  $q$  and the principal will know with certainty that one of the agents deviated. Assuming that the principal's punishments are strong enough, then both agents can be made to do what they are supposed to do, at no cost to the principal. All that remains is a standard-looking moral-hazard problem on the initial effort  $a$ . This example is similar to the coordination example in Section 4.2. Rotation sets up a two-person game on each project, and the resulting Nash equilibrium make deviations relatively easy to prevent. Indeed, the second-stage portion of the problem is closer to the team-production models discussed at the end of Section 4.2, in that the principal knows with certainty if someone deviated but does not know who.

Perfect inference is possible in this example because of the deterministic production function. For more general production functions, where inference is less than perfect, the analysis is less clear. Forces like those studied in Section 4.1 would push towards no-switching assignments. Limits on communication could work in a similar direction. Indeed, probabilistic rotation may be optimal. In risk neutral environments, both Hirao (1993) and Lewis and Sappington (1997) take steps in this direction. Hirao (1993) limits herself to two interim states and two outputs. She finds a condition relating disutility of effort that determines whether there will be switching. Lewis and Sappington (1997) also have two states but more outputs, linear disutility, and a different production function. If there are no costs to switching, they find that it is always optimal to separate the two production steps. Our analysis as well as these two papers suggests then that in the general model the optimality of switching depends a great deal on the parameters.

## 6 Conclusion

This paper is explicitly motivated by observations of job rotation and other intertemporal movements of workers within an organization. The forces for rotation that we analyzed include information revelation, scrambling of information, and coordination properties of the production function. These features are complementary to other stories like that of learning (Meyer (1994)) or commitment (Ickes and Samuelson (1987)). Consequently, distinguishing the models in data might be difficult, particularly if more than one factor was at work. Nevertheless, some comparisons could probably be made. For example, studies of the quality of managerial reporting systems could be made to determine if rotation improves the accuracy or usefulness of managerial reports. In the learning story of Meyer (1994), an organization consists of overlapping generations of workers who engage in team production. The precise assignment of workers to teams affects the ability of the organization to make inferences about worker quality. To make any sort of inference the organization needs each team to finish its project so rotation occurs after projects are finished. In contrast, rotation in this paper is done during a project, before it finishes. The timing of rotation in the data might help to disentangle these different effects.

More generally some workers are better at certain activities than others. Generating better matches is a classic reason for worrying about assignments. Indeed, the working paper version of this paper (Prescott and Townsend (2003)) incorporates private information into the classical assignment model of Koopmans and Beckmann (1957), generating an additional reason for job rotation.

## A Proofs

**Proposition 5** *If  $f(a, b)$  satisfies Assumption 1 then the best switching contract strictly dominates the best no-switching contract.*

**Proof:** We start with an optimal solution to the no-switching problem  $(a, b, c(q_1))$ . Because of the symmetry assumption it is characterized by  $a = b$ . Now consider the following switching contract. The agent is switched with probability one, effort on project

one is  $a$ , effort on project two is  $b$ , and consumption is

$$c(q_1, q_2) = U^{-1}(0.5U(c(q_1)) + 0.5U(c(q_2))), \quad (27)$$

where  $c(q_1)$  and  $c(q_2)$  are the terms of the no-switch contract applied to both of the projects the agent works. When the two outputs differ, consumption is set to a level that gives the same utility as if the contract randomized over the two outputs in the no-switching contract. If  $q_1 = q_2$ , then  $c(q_1, q_2) = c(q_1)$ , that is, consumption is unchanged. This contract gives the same utility as the no-switching contract because

$$\begin{aligned} & \sum_{q_1} p(q_1|f(a, b)) \sum_{q_2} p(q_2|f(a, b))U(c(q_1, q_2)) \\ &= \sum_{q_1} p(q_1|f(a, b)) \sum_{q_2} p(q_2|f(a, b)) (0.5U(c(q_1)) + 0.5U(c(q_2))) \\ &= 0.5 \sum_{q_1} p(q_1|f(a, b))U(c(q_1)) + 0.5 \sum_{q_1} p(q_2|f(a, b))U(c(q_2)) \\ &= \sum_{q_1} p(q_1|f(a, b))U(c(q_1)). \end{aligned} \quad (28)$$

The last line holds because of the symmetry in the problem. Furthermore, because of concavity this contract gives the principal more utility than the no-switching contract does. Our strategy is to show that it is incentive compatible under the switching regime.

In the switching regime there is no need to worry upward deviations since as long as the other agent is taking the recommended effort, an upward deviation has no effect on  $f$  and only lowers the agent's utility. For downward deviations, we have from the no-switching incentive constraints

$$\begin{aligned} \sum_{q_1} p(q_1|f(a, b))U(c(q_1)) - V(a) - V(b) &\geq \sum_{q_1} p(q_1|f(\hat{b}, \hat{b}))U(c(q_1)) - V(\hat{b}) - V(\hat{b}) \\ &= \sum_{q_1} p(q_1|f(a, \hat{b}))U(c(q_1)) - V(\hat{b}) - V(\hat{b}), \\ \sum_{q_2} p(q_2|f(a, b))U(c(q_2)) - V(a) - V(b) &\geq \sum_{q_2} p(q_2|f(\hat{a}, \hat{a}))U(c(q_2)) - V(\hat{a}) - V(\hat{a}) \\ &= \sum_{q_2} p(q_2|f(\hat{a}, b))U(c(q_2)) - V(\hat{a}) - V(\hat{a}), \end{aligned} \quad (29)$$

for all  $\hat{a} \leq a$  and  $\hat{b} \leq b$ . Notice that the two equalities hold because  $f(a, \hat{b}) = f(\hat{b}, \hat{b})$  and  $f(\hat{a}, b) = f(\hat{a}, \hat{a})$  for  $\hat{a} \leq b$  and  $\hat{b} \leq a$ .

Equivalently, (29) is

$$\begin{aligned}
& \sum_{q_1} p(q_1|f(a, b)) \sum_{q_2} p(q_2|f(a, b))U(c(q_1)) - V(a) - V(b) \\
\geq & \sum_{q_1} p(q_1|f(\hat{a}, b)) \sum_{q_2} p(q_2|f(a, \hat{b}))U(c(q_1)) - V(\hat{b}) - V(\hat{b}), \\
& \sum_{q_1} p(q_1|f(a, b)) \sum_{q_2} p(q_2|f(a, b))U(c(q_2)) - V(a) - V(b) \\
\geq & \sum_{q_1} p(q_1|f(\hat{a}, b)) \sum_{q_2} p(q_2|f(a, \hat{b}))U(c(q_2)) - V(\hat{a}) - V(\hat{a}),
\end{aligned} \tag{30}$$

for all  $\hat{a} \leq a$  and  $\hat{b} \leq b$ . Now, adding the two equations in (30) together, dividing by two, and then using the substitution in (27) delivers

$$\begin{aligned}
& \sum_{q_1} p(q_1|f(a, b)) \sum_{q_2} p(q_2|f(a, b))U(c(q_1, q_2)) - V(a) - V(b) \\
\geq & \sum_{q_1} p(q_1|f(\hat{a}, b)) \sum_{q_2} p(q_2|f(a, \hat{b}))U(c(q_1, q_2)) - V(\hat{a}) - V(\hat{b})
\end{aligned}$$

for all  $\hat{a} \leq a$ ,  $\hat{b} \leq b$ . This equation is the incentive constraint for downward deviations in the switching regime.

The constructed switching contract is feasible because it is incentive compatible and satisfies the participation constraint. Furthermore, it increases the principal's utility. Therefore, the optimal switching contract is better than the best no-switching contract. **Q.E.D.**

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