

## REFERENCES

- Arrow, K. J. (1971) *Essays in the Theory of Risk-Bearing* (Chicago: Markham).
- Attanasi, E. D. and S. R. Johnson (1975) 'Expectations, Market Structure, and Sequential Bid Pricing', *Southern Economic Journal*, 42: 18–32.
- Attanasi, E. D. and S. R. Johnson (1977) 'Sequential Bidding Models: A Decision Theoretic Approach', *Industrial Organization Review*, 14: 234–45.
- Baron, D. P. (1972). 'Incentive Contracts and Competitive Bidding', *American Economic Review*, 62: 384–94.
- Engelbrecht-Wiggans, R. (1980) 'Auctions and Bidding Models: A Survey', *Management Science*, 25: 1272–7.
- Friedman, L. (1956) 'A Comparative Bidding Strategy', *Operations Research*, 4: 104–12.
- Harris, M. and A. Ravid (1981) 'Allocation Mechanisms and the Design of Auctions', *Econometrica*, 49: 1477–99.
- Harsanyi, J. C. (1967–1968) 'Games With Incomplete Information Played by "Bayesian" Players', *Management Science*, 14: 159–82; 15: 320–334, 486–502.
- Holt, C. A. Jr. (1980) 'Competitive Bidding for Contracts Under Alternative Auction Procedures', *Journal of Political Economy*, 88: 433–45.
- Kortanek, K. O., J. V. Soden and D. Sodaro (1973) 'Profit Analysis and Sequential Bid Price Models', *Management Science*, 20: 396–417.
- Kuhlman, J. M. and S. R. Johnson (1983) 'The Number of Competitors and Bid Prices', *Southern Economic Journal*, 50: 213–20.
- Milgrom, P. R. (1981) 'Rational Expectations, Information Acquisition, and Competitive Bidding', *Econometrica*, 49: 921–44.
- Milgrom, P. R. and R. J. Weber (1982) 'A Theory of Auctions and Competitive Bidding', *Econometrica*, 50: 1089–1122.
- Pratt, J. W. (1964) 'Risk Aversion in the Small and in the Large', *Econometrica*, 32: 122–36.
- Vickrey, W. (1961) 'Counterspeculation, Auctions, and Competitive Sealed Tenders', *Journal of Finance*, 16: 8–37.
- Wilson, R. (1967) 'Competitive Bidding With Asymmetrical Information', *Management Science*, 13: A816–A820.
- Wilson, R. (1969) 'Competitive Bidding With Disparate Information', *Management Science* 15: 446–8.
- Wilson, R. (1977) 'A Bidding Model of Perfect Competition', *Review of Economic Studies*, 4: 511–18.
- Wilson, R. (1979) 'Auctions of Shares', *Quarterly Journal of Economics*, 93: 675–98.

## 22 Taking Pure Theory to Data: Arrow's Seminal Contribution

Robert M. Townsend\*

It is sometimes thought that the Arrow-Debreu model is a strange if not dubious starting point for empirical work. This short note honouring Arrow's (1953) seminal contribution takes the opposite point of view. It argues that the Arrow-Debreu model is rich in empirical implications, both directly, on its own, and indirectly, as the fountainhead of contributions that seek to explain otherwise anomalous observations.

To review briefly, one of the key insights of Arrow (1953) and Debreu (1959) was that the standard general equilibrium model could be easily modified to incorporate uncertainty. Essentially, one need only expand the commodity space by indexing all commodities to states of nature, publicly observed realizations of the random components of the model. Further, standard theorems on the existence and characterization of Pareto-optimal allocations and on the existence and optimality of competitive equilibria, of Arrow, Debreu, McKenzie and others, follow naturally in this framework. This insight, then, gave birth to two complementary contributions. The first is an incredibly powerful analytic method. The second is a systematic way of ordering observations. I shall take up each of these in turn.

The analytic method is so powerful that it is now standard in much of the profession. In teaching students of economics about the possibilities of risk sharing, for example, it is common to present them with an Edgeworth box diagram with two state-contingent commodities. One then emphasizes that a Pareto-optimal allocation has the property, following Hicks, and barring exceptional cases, that marginal rates of substitution across these two commodities should be equated. More generally, this kind of state-space analysis underlies the subsequent contributions of Wilson (1968) and others in the determination of

\*As will have become evident, Arrow's work has had a deep influence on my own research. In fact, a more personal interpretation of this chapter is that it documents that influence, hence the frequent references to my own work.

Pareto-optimal risk-sharing arrangements, *vis à vis* concave programming problems. One might also go further in such set-ups, allowing representative individuals to buy and sell in competitive markets contingent claims on consumptions, taking (market-clearing) prices as given. Indeed, following Arrow (1953), securities are naturally viewed as bundles of such contingent claims. Then, with non-trivial production possibilities, earlier Modigliani-Miller invariance results on the value of the firm follow immediately. The point, again, is that the state-space approach is a useful way to conduct analysis in economic models.

The second, complementary contribution of the Arrow (1953)-Debreu (1959) model, and more to the point of this chapter, is that it is rich in empirical implications. Since this point may be in contention, some elaboration on the method of economic science seems necessary. The view adopted here is essentially that expounded by Lucas (1980), that a model is an experimental laboratory. We, the modellers or experimenters, specify the endowments, preferences, and technology available to agents of the model, the subjects of the experiment, as it were, and then we attempt to predict how the agents will behave. A fundamental tenet for single-agent models is that the single agent will attempt to do as well as possible for himself under the specified endowments, preferences, and technology, much like Robinson Crusoe.

Multi-agent models, as Lucas notes, are more complicated, requiring in addition some specified form of interaction or some premise as to the outcome of this interaction. For example, we might suppose with Lucas that the outcome is necessarily the one that would be achieved in competitive markets, or, alternatively, that the outcome be in the core, or, more weakly, that the outcome be Pareto-optimal. But the point is that, in conjunction with the maximization hypothesis, any such premise delivers (in principle) a well-defined mapping from endowments, preferences, and technology into actions and allocation. It is thus that a theory can have empirical content.

It is in this way, then, that the possibility of exchanging contingent claims on consumption is transformed from a theoretical and analytically powerful insight into a definite, fundamental prediction about what we should see happening in actual economies. In particular, the theory predicts that, except for what would seem to be very special cases, rather strong alignments of preferences and endowments, there will definitely be some sharing of exogenous risk, some agreement to smooth consumption across individuals relative to autarky. In the Edgeworth-box economy described earlier, for example, assuming

common beliefs and no shocks to preferences, each of the two individuals would share some (variable) fraction of the social pie, the sum of their endowments. Since versions of this should be true more generally, beyond the confines of the two-agent economy, one might predict that risk-sharing arrangements would be pervasive in actual economies. Further, if one is willing to take a stand on individual preferences, perhaps up to specified parameters, then it seems that the theory can be fitted to actual data, that is, to series on individual and aggregate consumption. This idea has been forwarded by José Scheinkman (1984) and a student at Chicago, Paulo Leme (1984).

It is in conjunction with the supposed operation of competitive markets, however, that contingent claims theory has been found wanting empirically. As Arrow (1953) noted in his original contribution, we do not seem to see individuals trading claims on consumption, claims that are indexed by all possible states of the world. And so, on the face of it, the theory just fathered was in immediate jeopardy. But Arrow (1953) went on to point out that not all these contingent commodities would be needed, that with spot markets it would be enough for individuals to trade securities at an initial date; securities denominated in nominal terms and promising nominal payoffs contingent on the state of nature. Since we do observe the existence of such securities, it is certainly not obvious that the theory is invalid. In fact, as Arrow and others have pointed out, it is enough that returns on a given set of securities span the space of all possible returns. For example, as Townsend (1978) has shown, even the non-contingent claims on commodities associated with forward commodity markets can have the spanning property under rather general circumstances. It is thus that the theory proves useful in ordering observations and, ultimately, in providing a framework for policy analysis.

Still, all in all, the idea persists that securities and commodity markets are incomplete. Perhaps one reason for this is that the theory only makes predictions about what would happen if states of the world were publicly observable, whereas, as Radner (1968) and others have emphasised, in actual economies, private information would seem to be pervasive. Indeed, if contracts in such economies were limited to publicly observed states, as Radner suggests, the attainable consumption set of individuals would be more limited than what might be suggested *a priori* by the results of the spanning literature. For example, in Townsend (1979) and Baiman and Demski (1980), costly verification of privately-observed states can make formal contracts quite limited, relative to the possibilities suggested by nature. The point, then, is that

we begin to see here the productive role of the Arrow (1953) contribution not only in ordering observations but also in guiding us to better theory.

In fact, the incorporation of private information into the Arrow-Debreu general equilibrium model has led again to two complementary contributions, one analytic and the other tied to ordering observations. Again, these will be discussed in turn.

Many of the earlier efforts of contract theory and game theory which incorporated private information were brought together, made formal, and forwarded in the papers of Harris and Townsend (1981) and Myerson (1979, 1982) on resource allocation mechanisms, following in the spirit of the earlier work of Hurwicz (1972). In these papers, the indexation insight of Arrow and Debreu was applied rather ruthlessly, so that, in the end, commodities could still be tied (potentially) to states of the world, even though the states were privately observed. The key result that allowed this outcome is termed the 'revelation principle': that without loss of generality certain incentive compatibility constraints can be imposed, so that individuals with private information are given an incentive to announce their information truthfully. The result, then, for a given economic model is a programming problem and access (potentially) to the same set of powerful analytic methods mentioned earlier. Further, as argued by Myerson (1979) and Prescott and Townsend (1984a, 1984b), for example, these programming problems can be made concave by the use of lotteries, again, an important analytic advantage. In any event, these programming methods underlie the analysis of the principal-agent paradigm, for example, and much recent work. Of course, in drawing a distinction between risk sharing and incentives, these papers give us a useful way to think about the world.

Regarding the ordering of observations directly, it seems fair to say that the incorporation of private information into the Arrow-Debreu general equilibrium paradigm has had some rather unexpected consequences. First, the theory predicts that in many circumstances the incentive compatibility constraints should preclude trade, consistent with earlier results, but that in other circumstances the indexation of commodities to private information should be non-trivial, with active exchange contingent on privately-announced outcomes, followed by stringent prohibitions on further exchange. Secondly, and again, the theory predicts the use of lotteries in some circumstances, allowing beneficial trade that would otherwise be precluded. And thirdly, in conjunction with the competitive market hypothesis, the theory, as

extended by Prescott and Townsend (1984a, 1984b), predicts a trade in commodities that seems to have no analogue in actual markets.

One is reminded then of scepticism regarding the usefulness of the original Arrow contribution. In fact, however, as Prescott and Townsend have argued, several of these strong predictions may find support in actual observations. In particular, a trade in commodities tied to individual announcements of privately-observed shocks can take the form of trade in contracts with options, options effected entirely by the individual at his own discretion, without verification of claimed events. Indeed, futures contracts have such options, as detailed examination reveals; delivery, for example, is sometimes made contingent on circumstances that only the shipper might know. And on reflection it seems that many labour contracts have similar options. The use of lotteries, however, remains problematical, but Prescott and Townsend do argue that queues and first-come-first-served allocation devices may induce the requisite artificial risk predicted by the theory. The point, again, is that the theory predicts the use of options and lotteries as mutually beneficial arrangements. These are indeed the arrangements that maximizing, self-interested agents would come to in worlds with uncertainty and private information. Thus, we should either find these arrangements in actual situations or search for elements missing from the theory. Either way, one hopes to make some progress.

There remains, of course, a variety of observations and institutions that cannot be explained with the Arrow-Debreu paradigm, even with the incorporation of private information. Perhaps the most prominent of these is the use of currency and written financial instruments. In fact, here we might part company with Arrow (1953) somewhat and note that his security arrangements were imposed exogenously; there is nothing in his theory that predicts the use over time of nominally denominated securities, as opposed to direct initial trade in contingent commodities themselves. Indeed, 'money' in this sense is inessential in the Arrow-Debreu paradigm, as such diverse authors as Brunner and Meltzer (1971), Cass and Shell (1983), Clower (1971), Hahn (1973), and Wallace (1980) have pointed out.

But the point of this note is that it is a virtue of the Arrow-Debreu paradigm that it delivers such strong implications patently inconsistent with the facts. In using it we are forced to seek additional key elements beyond uncertainty and private information, elements such as limited commitment and limited communication as in Townsend (1980), and Townsend (1985), respectively. Indeed, it is in drawing a distinction among these elements that we force analytic advances. In turn, as our

thinking is made more precise, we are better able to confront observations. In short, far from being devoid of empirical content, the Arrow-Debreu model illustrates economic science at its best.

## REFERENCES

- Arrow, K. J. (1953) 'Le Role des Valeurs Boursières pour la Repartition la Meilleure des Risques', *Econometrie: Colloques Internationaux du Centre National de la Recherche Scientifique*, 47-8.
- Baiman, S. and J. Demski (1980) 'Economically Optimal Performance Evaluation and Control Systems', *Journal of Accounting Research*, Supplement: 184-220.
- Brunner, K. and A. Meltzer (1971) 'The Uses of Money: Money in the Theory of an Exchange Economy', *American Economic Review*, 61: 784-805.
- Cass, D. and K. Shell (1983) 'Do Sunspots Matter?', *Journal of Political Economy*, 91.
- Clower, R. W. (1971) 'Theoretical Foundations of Monetary Policy', in G. Clayton, J. C. Gilbert and R. Sedwick (eds) *Monetary Theory and Policy in the 1970's* (Oxford: Oxford University Press).
- Debreu, G. (1959) *Theory of Value* (New Haven: Yale UP).
- Hahn, F. H. (1973) 'On the Foundations of Monetary Theory', in M. Parkin and A. R. Nobay (eds) *Essays in Modern Economics* (New York: Harper & Row).
- Harris, M. and R. M. Townsend (1981) 'Resource Allocation under Asymmetric Information', *Econometrica*, 49: 33-69.
- Hurwicz, L. (1972) 'On Informationally Decentralized Systems', in C. B. McGuire and R. Radner (eds) *Decision and Organization* (Amsterdam: North Holland).
- Leme, P. (1984) 'Integration of International Capital Markets', manuscript (University of Chicago).
- Lucas, R. (1980) 'Methods and Problems in Business Cycle Theory', *Journal of Money, Credit and Banking*, 12.
- Myerson, R. (1979) 'Incentive Compatibility and the Bargaining Problem', *Econometrica*, 47.
- Myerson, R. (1982) 'Optimal Coordination Mechanisms in Generalized Principal-Agent Problems', *Journal of Mathematical Economics*, 10: 67-81.
- Prescott, E. C. and R. M. Townsend (1984a) 'Pareto Optima and Competitive Equilibria with Adverse Selection and Moral Hazard', *Econometrica*, 52: 21-46.
- Prescott, E. C. and R. M. Townsend (1984b) 'General Competitive Analysis in an Economy with Private Information', *International Economic Review*, 25: 1-20.
- Radner, R. (1968) 'Competitive Equilibrium under Uncertainty', *Econometrica*, 36: 31-58.
- Scheinkman, J. (1984) 'General Equilibrium Models of Economic Fluctuations: A Survey of Theory', manuscript (Univ. of Chicago).
- Townsend, R. M. (1978) 'On the Optimality of Forward Markets', *American Economic Review*, 68: 54-66.
- Townsend, R. M. (1979) 'Optimal Contracts and Competitive Markets with Costly State Verification', *Journal of Economic Theory*, 21: 265-93.
- Townsend, R. M. (1980) 'Models of Money with Spatially Separated Agents', in J. H. Kareken and N. Wallace (eds) *Models of Monetary Economies* (Federal Reserve Bank of Minneapolis).
- Townsend, R. M. (1985) 'Economic Organization with Limited Communication', manuscript (Univ. of Chicago).
- Wallace, N. (1980) 'The Overlapping Generations Model of Fiat Money', in J. H. Kareken and N. Wallace (eds) *Models of Monetary Economies* (Federal Reserve Bank of Minneapolis) pp. 49-78.
- Wilson, R. (1968) 'The Theory of Syndicates', *Econometrica*, 36: 119-32.