Identification and Falsification with Data

14.04 Intermediate Micro Theory: Lecture 21

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Content of Economic Theory

- Test with as little structure as possible

Outline

- Consumer Optimization and General Equilibrium theory
- A Unified Approach - Infinite Data and the Slutzky Matrix
- Finite data and Reveled preference Axioms
- Computational considerations
- Testing GE theory with Finite Data
- Falsifiability

The next two propositions gives us some basic properties of the expenditure function e and the Hicksian demand h.

Proposition (Properties of the expenditure function: MWG Prop. 3.E.2)

Suppose that $u(.)$ is a continuous utility function representing a locally nonsatiated preference relation $\succsim$ defined on the consumption set $X = \mathbb{R}^L_+$. Then, the following statements hold:

1. *$e(p, u)$ is homogeneous of degree one in p, and $h(p, u)$ is homogeneous of degree 0 in p.*
2. *$e(p, u)$ is strictly increasing in u and nondecreasing in p_l for any l*
3. *$e(p, u)$is concave in p (for fixed u)*
4. *$e(p, u)$ is continuous in (p, u)*

Proposition (Properties of Hicksian demand: MWG Prop. 3.E.3, 3.G.1, 3.G.2)

Suppose u is continuous and locally non-satiated. Then

1. *(No excess utility) If* $x \in h(p,u) \implies u(x) = u$
2. *If* $\succsim$ *are convex, then* $h(p,u)$ *is convex valued. If preferences are strictly convex, then* $h(p,u)$ *is single valued and continuous.*
3. *If* $h(p,u)$ *is single valued, then* $e(p,u)$ *is differentiable, and moreover*

$$\frac{\partial e(p,u)}{\partial p_l} = h_l(p,u) \ \text{for all } l = 1, 2, ..., L \tag{8}$$

4. *If h is differentiable, then the Jacobian matrix with respect to prices;* $\nabla_p h(p,u)$ *(a* $L \times L$ *matrix) satisfies:*

$$\nabla_p h(p,u) = \nabla_p^2 e(p,u) \tag{9}$$

a symmetric, negative semidefinite matrix

Slutsky's Decomposition

- We have a testable implication of utility maximization from part of a consumer: the Jacobian matrix of the Hicksian Demand function should be symmetric and negative semidefinite.
- However, even if we have infinite data about the consumer's choices, we can only observe the Walrasian Demand Function $x(p, w)$.
- In the next proposition, we will express $\nabla_p h(p, w)$ in terms of the Walrasian demand function and its derivatives, and therefore, in **terms of observables**.
- Given $(p, w) \gg 0$, define the Slutsky's **matrix** $\mathbf{s}\,(p, w)$ (of dimensions $L \times L$) with generic element $\mathbf{s}_{lk}\,(p, w)$ as

$$\mathbf{s}_{lk}\,(p, w) \equiv \frac{\partial x_l\,(p, w)}{\partial p_k} + \frac{\partial x_l\,(p, w)}{\partial w} x_k\,(p, w) \text{ for all } l, k = 1, 2, ..., L \tag{10}$$

See that $\mathbf{s}_{lk}(p, w)$ can be calculated only by knowing the Walrasian demand function $x(p, w)$. The next proposition tells us that the Jacobian of the Hicksian demand is exactly the Slutsky's matrix.

Proposition (Slutky Equation, MWG Prop. 3.G.3)

Suppose $u(.)$ is a continuous utility function representing a locally non-satiated and strictly convex preference relation $\succsim$ on $X = \mathbb{R}^L_+$. Then, for all $(p, w) \gg 0$ and $u = v(p, w)$ we have that

$$\frac{\partial h_l(p, u)}{\partial p_k} = \mathbf{s}_{lk}(p, w) \tag{11}$$

or in matrix notation:

$$\nabla_p h(p, u) = \mathbf{s}(p, w) \text{ with } u = v(p, w) \tag{12}$$

Proof.

From equation (6) of the previous lecture, duality

$$x_l\left[p, e(p,u)\right] = h_l(p,u) \text{ for all } (p,u) \Longrightarrow$$

$$\frac{\partial\left\{x_l\left[p, e(p,u)\right]\right\}}{\partial p_k} = \frac{\partial h_l(p,u)}{\partial p_k}$$

$$\underset{(i)}{\Longleftrightarrow} \frac{\partial x_l\left[p, e(p,u)\right]}{\partial p_k} + \frac{\partial x_l\left[p, e(p,u)\right]}{\partial w}\frac{\partial e(p,u)}{\partial p_k} = \frac{\partial h_l(p,u)}{\partial p_k}$$

$$\underset{(ii)}{\Longleftrightarrow} \underbrace{\frac{\partial x_l\left[p, e(p,u)\right]}{\partial p_k}}_{\frac{\partial x_l(p,w)}{\partial p_k} \text{ by } (6)} + \underbrace{\frac{\partial x_l\left[p, e(p,u)\right]}{\partial w}}_{\frac{\partial x_l(p,w)}{\partial w} \text{ by } (6)} \underbrace{h_k(p,u)}_{= x_k(p,w) \text{ by } (5)} = \frac{\partial h_l(p,u)}{\partial p_k} \underset{(iii)}{\Longleftrightarrow}$$

$$\frac{\partial x_l(p,w)}{\partial p_k} + \frac{\partial x_l(p,w)}{\partial w} x_k(p,w) = \frac{\partial h_l(p,u)}{\partial p_k}$$

Using in (i) the Chain rule to differentiate with respect to p_k, in (ii) Property 3 in Proposition 4 (of the previous lecture, statements of duality) and in (iii) the identities between expenditure and value functions, and between Walrasian and Hicksian demands. In sum, we can equate (10) and (11). □

The most important Corollary of this Proposition is a restriction on the observable Slutsky matrix $\mathbf{s}(p, w)$

Corollary (Testable Restrictions on $\mathbf{s}(p, w)$)

Suppose $u(.)$ is a continuous utility function representing a locally non-satiated and strictly convex preference relation $\succsim$ on $X = \mathbb{R}^L_+$. Then,

$$\mathbf{s}(p, w) \text{ is symmetric and negative semidefinite for all } (p, w) \gg 0 \tag{13}$$

Proof.

From Claim 4 of Proposition (4) we have that $\nabla_p h(p, u) = \nabla^2_p e(p, u)$ for all (p, u). From Proposition (5) we have that $\nabla_p h(p, u) = \mathbf{s}(p, w)$ with $u = v(p, w)$. Therefore, putting these two results together, we have that for all $(p, w) \gg 0$:

$$\mathbf{s}(p, w) = \nabla^2_p e[p, v(p, w)]$$

and because e is concave in p, this means that $s(p, w)$ must therefore be a symmetric and negative semidefinite matrix for all $(p, w) \gg 0$. □

The Corollary gives us a way of testing whether preferences are rational, convex and locally non-satiated: suppose we have infinite data on some consumer optimization problem, so that we can actually observe her individual Walrasian demand function $x(p, w)$ entirely. This implies that we can observe not only the levels of the Walrasian demand, but also its derivatives, so we can derive explicitly the Slutsky matrix $\mathbf{s}(p, w)$.

Consider the following hypothesis about behavior:
Hypothesis: The demand function $x(p, w)$ is the result of consumer maximization from some continuous, locally non-satiated and strictly quasi-concave utility function $u : \mathbb{R}^L_+ \to \mathbb{R}$ over budget sets $B_{p,w} = \left\{x \in \mathbb{R}^L_+ : px \leq w\right\}$

Thesis: Corollary (6) implies that the Slutsky matrix $\mathbf{s}(p, w)$ is symmetric and negative semidefinite for all $(p, w) \gg 0$

Therefore, the **hypothesis is Falsifiable**: if we find a pair $(\widehat{p}, \widehat{w})$ such that $\mathbf{s}(\widehat{p}, \widehat{w})$ is either not symmetric or not negative semidefinite, then the demand function $x(p, w)$ could not have come from a consumer maximizing utility. However, if we such a pair does not exist, then we cannot reject the hypothesis: the data is observationally equivalent to coming from a consumer maximizing her utility. Note that even with infinite data, the hypothesis is testable. More specifically, we propose the following test for rationality:

Test: Look for a pair of price and income $(\widehat{p}, \widehat{w})$ such that $\mathbf{s}(\widehat{p}, \widehat{w})$ is either not symmetric or not negative semidefinite. If such a pair exists, reject hypothesis of Rationality. If not, do not reject

Lets see which properties we can use to test whether a demand function $x(p, w)$ came from a consumer maximizing some utility function $u : \mathbb{R}^L_+ \to \mathbb{R}$

1. **(Homogeneity of degree 0)** For any rational preference with single-valued demand function $x(p, w)$ we must have that x is homogeneous of degree 0. Therefore, *if there exist some* (p, w) *and some* $\alpha > 0$ *such that* $x(\alpha p, \alpha w) \neq x(p, w)$ *, then the demand function* $x(p, w)$ *could not come from rational preferences with a single valued demand fun*
2. **(Walras Law** If preferences are locally nonsatiated, then we must have that $px(p, w) = w$ for all (p, w). Therefore, *if* $\widehat{p}x(\widehat{p}, \widehat{w}) < \widehat{w}$, *for some* $(\widehat{p}, \widehat{w})$, *then the demand could not come from locally non-satiated preferences*
3. **(Slutsky Matrix)**If the demand function is differentiable (something that we can check) and single valued, then $s(p, w)$ is symmetric and negative semidefinite. Therefore, *if there exist* $(\widehat{p}, \widehat{w})$ *such that* $s(\widehat{p}, \widehat{w})$ *is not symmetric or is not negative semidefinite, then preferences could not come from locally non-satiated preferences*

See that neither of these 3 tests can tell us whether the underlying preferences were convex or not. Note that the test with the most content is the third one, since typically any consumer (whether maximizing utility or not) would satisfy the first two conditions. Moreover, we do not know (yet) if there are more tests that we could device to test the hypothesis of rationality.

However, and amazingly enough, the result is negative: the list of "tests" we have provided on $x(p, w)$ is exhaustive: that means, for any Walrasian demand function $x(p, w)$ that satisfies homogeneity of degree 0, Walras Law and Slutzky, we can create a candidate utility function that would generate $x(p, w)$ as a
(single valued) Walrasian Demand function. This result is called "Integrability".

Theorem (Integrability Theorem (Jehle and Reny, Thm 2.6))

The following statements hold:

(a) : *If preferences of a household are given by a utility function* $u : \mathbb{R}^L_+ \to \mathbb{R}$ *that is strictly increasing, and strictly quasiconcave, then the Walrasian demand function*

$$x(p, w) \equiv \arg \max_{x \in B_{p,w}} u(x)$$

Satisfies Homogeneity of degree 0, Walras law and its Slutsky matrix $\mathbf{s}(p, w)$ *is symmetric and negative semidefinite for all* (p, w) *.*

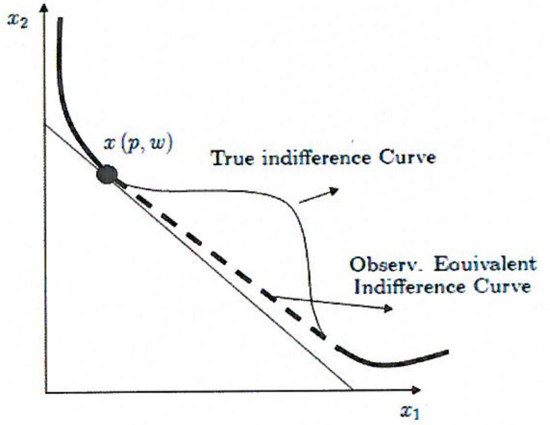
(b) : *Take a demand function* $x(p, w)$ *that is differentiable with continuous derivatives, and satisfies:*

1. *Homogeneity of degree 0*
2. *Walras law*
3. *It slutsky matrix* $\mathbf{s}(p, w)$ *is symmetric and negative semidefinite for all* (p, w)

Then, there exist a utility function $u : \mathbb{R}^L_+ \to \mathbb{R}$ *that is increasing, quasiconcave utility function such that*

$$x(p, w) \in \arg \max_{x \in B_{p,w}} u(x) \ \textit{for all} \ (p, w) \tag{1}$$

Non-testability of Convex Preferences



In this figure, the true underlying indifference curve comes from a non-convex preference ordering on $\mathbb{R}^2_+$. However, the agent will never choose bundles on "non convex sections" of the indifference curve: she will only choose from the sections in thick black lines. Therefore, we can "complete" the indifference curves in such a way that the underlying preferences would seem to be convex, even though the true underlying preferences may not be so.

Some economists takes this result as a reason for assuming convex preferences: if we are going to model consumer behavior in competitive markets, whether preferences are convex or not is irrelevant, since the data generated will be observationally equivalent as an alternative model in which consumers have convex preferences.

Testing Individual Rationality with Finite Data: Introduction

In the previous section on testing consumer rationality, we assumed that we could observe the entire Walrasian Demand function $x(p, w)$ (so we can observe infinite data on all the possible decisions on all pairs of prices and income (p, w)). Based on that assumption, we concluded that we could test whether the Walrasian Demand came from an (UMP) problem with locally non-satiated preferences.

However, in real life we never have infinite data. Typically, an observer can only have access to a finite set of decisions taken by some consumer (for example, in a lab experiment setting). Clearly, the test about the Slutsky matrix (which depended on infinitesimal variations of the Walrasian demand, since it is defined from derivatives of the Walrasian Demand function) is of no use here. The question we ask then is whether we can construct restrictions on the data such that we can test the Hypothesis of Rationality. It turns out that even with finite data, we can find tests that are not only necessary but also sufficient to test the hypothesis of rationality. This is the topic of this section.

Weak Axiom of Revealed Preference (Cont.)

Definition (Weak Axiom of Revealed Preference (WARP))

Let (p^1, x^1) and (p^2, x^2) be a pair of observations on prices and consumption bundles, with $p^t \gg 0$ and $x^t \geq 0$ for $t = 1, 2$ and such that $(p^1, x^1) \neq (p^2, x^2)$. We say that individual choices (given by x^i) satisfy the Weak Axiom of Revealed Preference if, whenever $p^2 x^1 < p^2 x^2$, we must then have that $p^1 x^2 > p^1 x^1$.

The idea of this definition is fairly intuitive: take the price vector p^2; at those prices, the consumer chooses the bundle x^2. However, x^1 was also available at those prices (i.e. $p^2 x^1 \leq p^2 x^2$), but was not chosen. This implies that the consumer preferred x^2 over x^1 (so, **she revealed her preference**). Therefore, it must be the case that when prices were p^1, x^2 could not be available for consumption, because if that were the case, then she would again choose x^2 instead of x^1. Figure 4 illustrates an example in which the WARP is **NOT** satisfied.

Weak Axiom of Revealed Preference (Cont.)

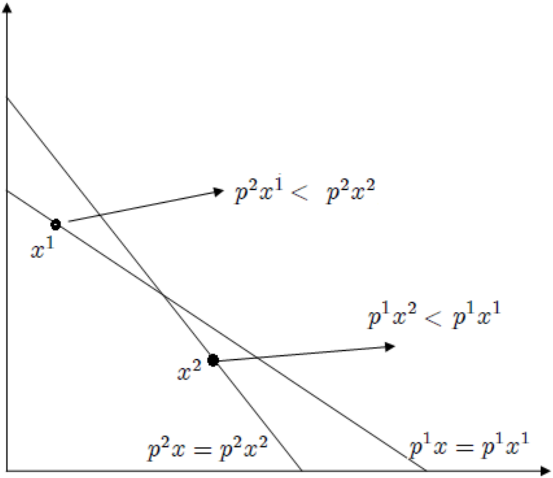


Figure 4: Failure of the Weak Axiom of Revealed Preference

Weak Axiom of Revealed Preference (Cont.)

Proposition

[Necessity of WARP] Suppose an agent has preferences $\succsim$ over $X \subseteq \mathbb{R}_+^L$, which are locally nonsatiated and rational. Then, for any two observations (p^1, w^1, x^1) and (p^2, w^2, x^2) we must have that (p^1, x^1) and (p^2, x^2) satisfy WARP.

The idea of the proof can be easily seen in Figure 5. See that if WARP is violated, as in Figure 4 then the indifference curves of the agent would intersect, which is impossible.

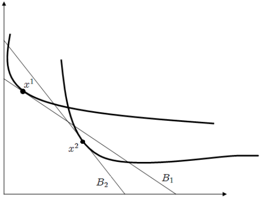


Figure 5: Idea of Proposition 10

GARP

Definition (Direct Revealed Preference)

We say that x^t is **Directly Revealed Preferred** to x^s, and we write "$x^t R^D x^s$" if $p^t x^s \leq p^t x^t$

Definition (Revealed Preference)

We say that x^t is (indirectly) **Revealed Preferred** to x^s, and we write "$x^t R x^s$", if there exist observations $\{(p^{r_j}, x^{r_j})\}_{j=1}^{k}$ such that

$$x^t R^D x^{r_1}, x^{r_1} R^D x^{r_2}, .., x^{r_k} R^D x^s$$

Definition (Generalized Axiom of Revealed Preference (GARP))

The observed data $\left\{\left(p^t, x^t\right)\right\}_{t=1}^{T}$ satisfies the General Axiom of Revealed Preference (GARP) if and only if

$$x^t R x^s \text{ implies } p^s x^t \geq p^s x^s \text{ for all } t, s \in \{1, 2, ..., T\}$$

"A Revealed Preference Approach to Computational Complexity in Economics", Echenique et al (2013)

Consumption theory assumes that consumers posses infinite computational abilities. Proponents of bounded rationality want to instead require that any model of consumer behavior incorporate computational constraints. In this paper, we establish that this requirement has no testable implications. Any consumption data set that is compatible with a rational consumer is also compatible with a rational and computationally bounded consumer (such a data set is rationalizable by a utility function that is easy to maximize over any budget set; specifically with a utility that can be maximized in strongly polynomial time).

The result is extended to data on multiple agents interacting in a market economy. We present sufficient conditions on observed market outcomes such that they are compatible with an instance of the model for which Walrasian equilibrium is easy to compute. Our result motivates a general approach for posing questions about the empirical content of computational constraints: the revealed preference approach to computational complexity. The approach complements the conventional worst-case view of computational complexity in important ways, and is methodologically close to mainstream economics.

CS critique of positive economics:

Economics is flawed because it assumes agents/society solve hard problems.

> *"As rational as consumers can possibly be, it is unlikely that they can solve in their minds problems that prove intractable for computer scientists equipped with the latest technology." – Gilboa, Schmeidler & Postlewaite*

> *"If an equilibrium is not efficiently computable, much of its credibility as a prediction of the behavior of rational agents is lost" – Christos Papadimitriou*

> *"If your laptop cannot find it, neither can the market" – Kamal Jain*

Methodological positivism.

CS (Bded. rationality) critique misunderstands the role of models in positive economics.

Model is a way of thinking about reality, i.e. about data.

Economic theory only states that reality behaves as if the theory is true.

Question: What is the empirical content of the hypothesis that consumers are boundedly rational (i.e. that they can't solve hard problems).

Answer: None.

Our Theorem: Given a consumption data set, the data is either not rationalizable at all, or it is rationalizable by a utility function that is easy to maximize.

The result is true even if there are indivisible goods.

Falsifiability of General Equilibrium: Brown and Matzkin (1996)

If we are given data on the number of households I in the economy, together with individual endowments $\{\omega_i\}_{i=1}^{I}$, the implications of non-testability of GE theory from the "Anything goes" theorem may fail. Now, the question the researcher wants to ask is:

> *"Given a set of observations on prices and individual endowments $\left\{p^t, \{\omega_i^t\}_{i=1}^{I}\right\}_{t=1}^{T}$, does there exist a sequence of* ***pure trade*** *economies $\mathcal{E}_t = \left\{\{\mathbb{R}_+^L, \succsim_i, \omega_i^t\}_{i=1}^{I}\right\}$ with aggregate excess demand function $z_{\mathcal{E}_t}\left(p^t\right)$ such that $z_{\mathcal{E}_t}\left(p^t\right) = 0$ for all $t = 1, 2, ..., T$?*

We now allow the data to come from different economies (they are indexed by t). The variability in the endowment vector data mechanically changes the economy. However, preferences are the same for all economies $\mathcal{E}_t$, and the source of variability is the different endowment profiles we might see in the data.

Aggregate Income is not Enough - Chiappori

Testable implications of general equilibrium theory: a differentiable approach, P. - A. Chiappori, I. Ekeland, F. Kübler, H. M. Polemarchakis, Journal of Mathematical Economics, February, 2004

Abstract: Is general equilibrium theory empirically testable? Our perspective on this question differs from the standard, Sonnenschein–Debreu–Mantel (SDM) viewpoint. While the SDM tradition considers aggregate (excess) demand as a function of prices, we suppose that what is observable is the equilibrium price vector as a function of the fundamentals of the economy. We apply this perspective to an exchange economy where equilibrium prices and individual endowments are observable. We derive necessary and sufficient conditions that characterize the equilibrium prices, as functions of initial endowments. Furthermore, we show that, if these conditions are satisfied, then the economy can generically be identified. Finally, we show that when only aggregate data are available, observable restrictions vanish. We conclude that the availability of individual data is essential for the derivation of testable consequences of the general equilibrium construct.

For instance, many micro studies in development, starting with Townsend's seminal investigation of risk sharing within an Indian village (Townsend, 1994), are based on data collected at the local level; it is not uncommon to observe endowments (say, individual crops) and prices within the village, a context to which our framework directly applies. Even in large economies, our result may still apply directly when individuals belong to a finite (and "small") number of homogeneous "classes". Finally, an interesting question is how our results can be extended to production economies. The idea is that, in a production context, changes in factor endowments will have an observable impact on factor prices, and that the corresponding equilibrium manifold can in principle be studied in a similar way. All this shall be the subject of further research.